

Parametric Resonance of Neutrino Oscillations in Electromagnetic Wave

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Experimental studies of solar, atmospheric and reactor neutrinos over the past several years provide almost certain indications that neutrinos oscillate, have masses and mix. These new properties of neutrinos, if confirmed by better statistics in the proceeding and forthcoming experiments, will require a significant departure from the Standard Model. It is well known [1] that massive neutrinos can have nonvanishing magnetic moment. For example, in the Standard Model supplied with $SU(2)$ -singlet right-handed neutrino the one-loop radiative correction generates neutrino magnetic moment which is proportional to neutrino mass. There are plenty of models [2] which predict much large magnetic moment for neutrinos.

In the light of new developments in neutrino physics, to get better understanding of the electromagnetic properties of neutrinos is an important task. In general, there are two aspects of this problem. The first one is connected with values of neutrino electromagnetic form factors. The second aspect of the problem implies consideration of influence of external electromagnetic fields, which could be presented in various environments, on neutrino possessing non-vanishing electromagnetic moments. The most important of the latter are the magnetic and electric dipole moments.

The majority of the previously performed studies of neutrino conversions and oscillations in electromagnetic fields deal with the case of constant transversal magnetic field or constant transversal twisting magnetic fields (see references [1-18] of paper [3]). Recently we have developed [3,4] the Lorentz invariant formalism for description of neutrino spin evolution that enables one to consider neutrino oscillations in the presence of an arbitrary electromagnetic fields. Within the proposed approach it becomes possible to study neutrino spin evolution in an electromagnetic wave and the new types of resonances in the neutrino oscillations in the wave field and in some other combinations of fields have been predicted.

The aim of this paper is to continue the study of the neutrino spin oscillations in the circular polarized electromagnetic wave. Here we consider the case when the wave amplitude is not constant but is periodically varying function

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of time and show for the first time that under certain conditions the parametric resonance of neutrino oscillations can occur in such a system. Note that in order to investigate this phenomenon we have to use the Lorentz invariant formalisms for neutrino evolution that have been developed in [3,4]. The possibility of the parametric resonance of neutrino flavour oscillations in matter with periodic variation of density was considered previously in [5]. It should be also pointed here that the conditions for a total neutrino flavour conversion in a medium consisting of two or three constant density layers were derived in [6]. For the recent discussion on the physical interpretations of these two mechanisms of increasing of neutrino conversion see refs. [7] and [8].

Let us consider evolution of a system $\nu = (\nu_+, \nu_-)$ composed of two neutrinos of different helicities in presence of a field of circular polarized electromagnetic wave with periodically varying amplitude. Here neutrinos ν_+ and ν_- correspond to positive and negative helicity states that are determined by projection operators

$$P_{\pm} = \frac{1}{2} \left(1 \mp \frac{(\vec{\sigma} \vec{p})}{|\vec{p}|} \right). \quad (1)$$

In general case it is important to distinguish helicity states $\nu = (\nu_+, \nu_-)$ and chirality states $\nu = (\nu_L, \nu_R)$. The latter are determined by the projection operators $P_{L,R} = (1 \mp \gamma_5)/2$. The evolution of such a system is given [3,4] by the following Schrödinger type equation

$$i \frac{\partial \nu}{\partial t} = H \nu, \quad H = \tilde{\rho} \sigma_3 + E(t) (\sigma_1 \cos \psi - \sigma_2 \sin \psi), \quad (2)$$

$$\tilde{\rho} = -\frac{V_{eff}}{2} + \frac{\Delta m^2 A}{4E}, \quad E(t) = \mu B(t) (1 - \beta \cos \phi).$$

Here the three parameters, $A = A(\theta)$ being a function of vacuum mixing angle, $V = V(n_{eff})$ being the difference of neutrino effective potentials in matter, and Δm^2 being the neutrino masses squared difference depend on the considered type of neutrino conversion process. The electromagnetic field is determined in the laboratory frame of reference by its frequency, ω , the phase at the point of the neutrino location, $\psi = g\omega t(1 - \beta/\beta_0 \cos \phi)$, ($g = \pm 1$), and the amplitude, $B(t)$, which is a function of time. The wave speed in matter could be less than the speed of light in vacuum ($\beta_0 \leq 1$), and ϕ is the angle between the neutrino speed $\vec{\beta}$ and the direction of the wave propagation. In the derivation of the Hamiltonian of eq.(2) terms proportional to $\frac{1}{\gamma^2} \ll 1$, $\gamma = (1 - \beta^2)^{-1/2}$, and also an oscillating function of time in the diagonal part are neglected (for details see [4]).

In order to study phenomenon of the parametric resonance of neutrino spin oscillations in such a wave we suppose that the amplitude $B(t)$ is given by

$$B(t) = B(1 + hf(t)), \quad (|h| \ll 1), \quad (3)$$

where $f(t)$ is an arbitrary function of time and h is a small dimensionless quantity of not fixed sign. It is convenient to introduce the evolution operator which determines the neutrino state at time t

$$\nu(t) = U(t)\nu(0)$$

if the initial neutrino state was $\nu(0)$. Using the Hamiltonian of eq.(2) we get the following equation for the evolution operator:

$$\dot{U}(t) = i[-\tilde{\rho}\sigma_3 + (-E_0 + \varepsilon f(t))(\sigma_1 \cos \psi - \sigma_2 \sin \psi)]U(t), \quad (4)$$

where $c\varepsilon = -E_0h$. In analogy with the case of the electromagnetic wave with nonvarying amplitude [3,4] the solution of eq.(4) can be written in the form

$$U(t) = U_{\vec{e}_3}(t)U_{\vec{l}}(t)F(t), \quad (5)$$

where $U_{\vec{e}_3}(t) = \exp(i\sigma_3 \frac{\psi t}{2})$ is the rotation operator around the axis \vec{e}_3 which is parallel with the direction of the neutrino propagation, and $U_{\vec{l}}(t) = \exp(i\vec{\sigma}\vec{l}t)$ is the rotation operator around the vector $\vec{l} = (-E_0, 0, \tilde{\rho} - \frac{\psi}{2})$. It should be noted that the solution of eq.(4) for the case of constant amplitude of the wave field ($\varepsilon = 0$) is given by the operator $U_0(t) = U_{\vec{e}_3}(t)U_{\vec{l}}(t)$.

From (4) and (5) it follows that the equation for the operator $F(t)$ is

$$\dot{F}(t) = i\varepsilon H_\varepsilon(t)F(t), \quad (6)$$

where

$$H_\varepsilon(t) = (\vec{\sigma}\vec{y}(t))f(t),$$

$$\vec{y} = (1 - 2\lambda_3^2 \sin^2 \Omega t, \lambda_3 \sin 2\Omega t, 2\lambda_1 \lambda_3 \sin^2 \Omega t),$$

and the unit vector λ is given by its components in the unit orthogonal basis $\vec{\lambda} = \lambda_1 \vec{e}_1 + \lambda_2 \vec{e}_2 + \lambda_3 \vec{e}_3 = \frac{\vec{l}}{\Omega}$, $\Omega = |\vec{l}|$. The detailed analysis of evaluation of the solution of eq.(6) can be found in [9]. Here we comment only on the main steps. Using the smallness of ε we expand the solution of eq.(6) in powers of this parameter:

$$F = \sum_{k=0}^{\infty} \varepsilon^k F^{(k)}, \quad (7)$$

where $F^{(0)} = \hat{1}$ is a unit matrix. For operators $F^{(k)}$ the recurrent formula is straightforward:

$$F^{(k+1)}(t) = i \int_0^t H_\varepsilon(\tau) F^{(k)}(\tau) d\tau. \quad (8)$$

Skipping further technical details to the first order in ε we get

$$F(t) = \hat{1} + i\varepsilon(\vec{\sigma}\vec{x}(t)) + O(\varepsilon^2), \quad (9)$$

where

$$\vec{x}(t) = \int_0^t \vec{y}(\tau) f(\tau) d\tau.$$

Thus for the probability of neutrino conversion $\nu_{i-} \leftrightarrow \nu_{j+}$ we get

$$P_{ij} = \left| \langle \nu_+ | U_{e3}(t) U_l(t) F(t) | \nu_- \rangle \right|^2 = \quad (10)$$

$$\lambda_1^2 \sin^2 \Omega t + 2\varepsilon \lambda_1 (x_1(t) \cos \Omega t + \lambda_3 x_2(t) \sin \Omega t) \sin \Omega t.$$

Note that Ω is nothing but the mean oscillation frequency of the neutrino system.

For further evaluation of solution of eq.(6) we have to specify the form of the function $f(t)$. As it is mentioned above, the purpose of this study is to examine the case when the parametric resonance for neutrino oscillations in electromagnetic wave with varying amplitude could appear. Having in mind a simple analogy (see, for example, [7]) between oscillations in neutrino system and oscillations of a classical pendulum [10], it is reasonable to suppose that the principal parametric resonance appears when the amplitude modulation function $f(t)$ is oscillating in time with frequency approximately equal to the twice mean oscillation frequency of the system. That is why we choose the function $f(t)$ to be

$$f(t) = \sin 2\Omega t \quad (11)$$

and get for the neutrino oscillations probability

$$P_{ij} = \left[\lambda_1^2 + \varepsilon \lambda_1 \lambda_3^2 t + \frac{\varepsilon \lambda_1}{\Omega} \left(1 - \frac{\lambda_3^2}{2} \right) \sin 2\Omega t \right] \sin^2 \Omega t. \quad (12)$$

It follows that in the case $\lambda_1 \varepsilon > 0$ the second term increases with increase of time t , so that the amplitude of the neutrino conversion probability may become close to unity. This is the effect of the parametric resonance in neutrino system in the electromagnetic wave with modulated amplitude that may

enhance neutrino oscillation amplitude even for rather small values of the neutrino magnetic moment μ and strength of the electromagnetic field and also when the system is far away from the region of ordinary spin (or spin-flavour) neutrino resonance.

In conclusion, let us consider the case when parameters of the neutrino system are far beyond the region of the ordinary resonance. Then the following condition is valid: $\lambda_1 \ll \lambda_3$, and the maximal neutrino conversion probability is small for the case of nonvarying ($h = 0$) amplitude of the electromagnetic field

$$P_{ijmax}(h = 0) = \frac{l_1^2}{l_1^2 + l_3^2} \ll 1. \quad (13)$$

If the amplitude of the field is modulated in accordance with eq.(11) the estimation for the critical time t_{cr} for which the probability P_{ij} could become close to unity for an arbitrary values of the neutrino mixing angle, masses, energy, and density of matter gives

$$t_{cr} \approx \frac{1}{\varepsilon n_1}. \quad (14)$$

It means that the parametric resonance enhancement of the neutrino oscillations occurs after neutrinos travel a distance

$$L = \frac{\Omega}{|h|[\mu B(1 - \beta \cos \phi)]^2}. \quad (15)$$

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